Linear Circuits

An introduction to electric circuit elements and a study of circuits containing such devices.

Dr. Bonnie H. Ferri
Professor and Associate Chair
School of Electrical and Computer Engineering

Concept Map: Background Module 1

Background
- Charge
- Current
- Voltage
- Power

Resistive Circuits
- Resistor
- Sources
- Circuits

Power

Reactive Circuits

Frequency Analysis
Charge and Current

Nathan V. Parrish
PhD Candidate & Graduate Research Assistant
School of Electrical and Computer Engineering

- Calculate the force charges exert on one another
- Calculate functions of charge and current

Lesson Objectives

- Calculate the forces two point charges exert on each other
- Calculate current based on a charge function
- Calculate charge based on a current function
Electric Charge

- Property of matter
- Quantized
- Measured in Coulombs (C)

\[ p^+ \quad e^- \]
\[ 1.602 \times 10^{-19} \text{ C} \quad -1.602 \times 10^{-19} \text{ C} \]

Electromagnetic Force

Coulomb’s Law:
\[ |F| = k_e \frac{|q_1 q_2|}{\gamma^2} \]
\[ k_e = \frac{1}{4\pi \varepsilon_0} \frac{\text{Nm}^2}{\text{C}^2} \]
\[ k_e = 8.988 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \]
Electric Fields

Electrical Current

Current

Units: ampere (A = C/s)

Variable: \( i \)

\[ i(t) = \frac{dq(t)}{dt} \]

\[ q(t) = \int_{t_0}^{t} i(\tau) d\tau + q(t_0) \]
Reference Directions

\[ i_1 = 1 \text{A} \quad i_2 = -1 \text{A} \]

Summary

- Discussed charge as a property of matter
- Calculated forces of charge using Coulomb’s Law
- Explored electric fields – the means of this interaction
- Described current as the time derivative of charge
- Emphasized the importance of current reference directions
Lesson Objectives

- Calculate voltage from the energy gained/consumed as a charge moves through an electric field
- Correctly specify voltages as references change
- Describe the operation of a chemical battery
- Identify if a battery is charging or discharging based on the voltage reference and current flow
Voltage

<table>
<thead>
<tr>
<th>Voltage</th>
<th>Units</th>
<th>volt (V = \frac{1}{C})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Variable</td>
<td>( v )</td>
</tr>
</tbody>
</table>

Origin of Voltage
Voltage Reference

Example: Finding Voltages

\[ V = \begin{cases} 6\text{V} & \text{if positive} \\ -6\text{V} & \text{if negative} \end{cases} \]

\[ V = \begin{cases} -4\text{V} & \text{if positive} \\ 3\text{V} & \text{if negative} \end{cases} \]
Lead-Acid Batteries - Discharging

Cathode: Reduction reaction
\[ \text{PbO}_2(s) + \text{SO}_4^{2-} (aq) + 4\text{H}^+(aq) + 2e^- \rightarrow \text{PbSO}_4(s) + 2\text{H}_2\text{O}(l) \]

Anode: Oxidation reaction
\[ \text{Pb}(s) + \text{SO}_4^{2-} (aq) \rightarrow \text{PbSO}_4(s) + 2e^- \]

Lead-Acid Batteries - Charging

Anode: Oxidation reaction
\[ \text{PbSO}_4(s) + 2\text{H}_2\text{O}(l) \rightarrow \text{PbO}_2(s) + 4\text{H}^+(aq) + \text{SO}_4^{2-} (aq) + 2e^- \]

Cathode: Reduction reaction
\[ \text{PbSO}_4(s) + 2e^- \rightarrow \text{Pb}(s) + \text{SO}_4^{2-} (aq) \]
Summary

- Charge creates electric fields
- Voltage is energy gained/released as charges move through an electric field
- Described how voltage originates from differences in charge density
- Case study: how lead-acid batteries work
Lesson Objectives

- Calculate power from energy function
- Calculate energy from a power function
- Use conservation of energy to find power of an unknown device
- Calculate power from voltage and current
- Find a voltage or a current for a device with a known power
**Power**

\[ p = \frac{dw}{dt} = \frac{dw}{dq} \frac{dq}{dt} = vi \]

**Energy**

<table>
<thead>
<tr>
<th>Units</th>
<th>joule (J)</th>
</tr>
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<tbody>
<tr>
<td>Variable</td>
<td>( w )</td>
</tr>
</tbody>
</table>

**Power**

<table>
<thead>
<tr>
<th>Units</th>
<th>watt (W=(\frac{1}{s}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
<td>( p )</td>
</tr>
</tbody>
</table>

\[ w = \int_{t_0}^{t} p(\tau) d\tau + w(t_0) \]

**Instantaneous Change**

\[ p = \frac{dw}{dt} = \frac{dw}{dq} \frac{dq}{dt} = vi \]

\[ w = \int_{t_0}^{t} p(\tau) d\tau + w(t_0) \]
Charging for Power

You run a power company. Do you charge customers for power or energy? Why?

Conservation of Energy

\[ \sum_{i=1}^{N} w_i = K \]
\[ \frac{d}{dt} \sum_{i=1}^{N} w_i = \frac{d}{dt} K = 0 \]
\[ \sum_{i=1}^{N} \frac{dw_i}{dt} = \sum_{i=1}^{N} p_i = 0 \]
Reference Direction

\[ p = iv \quad \quad p = -iv \]

Using Power for Analysis

\[ 120V \quad 60W \]

\[ i \]
Summary

- Described the relationship between power and energy and how to calculate them
- Described how voltage and current relate to power
- Presented a derivation for conservation of power and how this property is used in analysis
- Solved first simple analysis problem
Lesson Objectives

- Identify a set of circuit elements
- Identify the nodes in a circuit diagram
- Recognize self-contradictory circuits
- Identify open/closed circuit
- Identify when two circuits are equivalent
- Modify a circuit to form an equivalent circuit
Circuit Diagrams

What is Circuit Analysis?

- Devices are presented with some known parameters
- We wish to identify the behavior of the device
- Identify behavior based on the circuit using calculation

\[ G_1 = 0.005 \text{A/V} \]
\[ G_2 = 1000 \text{V/A} \]
Wires

- Zero internal resistance
- Nodes and junctions
- Zero voltage drop on node
- Arbitrary current possible

Independent Sources

- Independent Voltage Source
- Independent Current Source
Dependent Sources

- Dependent Voltage Source
- Dependent Current Source

Resistors

- Devices for resisting current
- Drawn as zig-zag lines
- Number of zigs ands zags can vary
- Other similar devices will be introduced as they are encountered
Ground

- Common Point of reference
- Path of electrons
  - Lightning rod (literal ground)
  - Prevent electrocution
  - Prevent damage to sensitive components

Open/Short Circuits
Self-Contradictory Circuits

5 V  1 A

Putting It Together

\( G_1 = 0.005 \text{A/V} \)
\( G_2 = 1000 \text{V/A} \)
Changing Circuits

Description of a node and how to identify one.
Introduction of independent and dependent voltage and current sources.
Introduction of a resistor – a device that will be discussed in more detail later.
Presentation of the idea of a ground.
Showed examples of self-contradictory circuits.
Described how circuits can be equivalent.

Summary
Module 1 Background Wrap Up

Dr. Bonnie H. Ferri
Professor and Associate Chair
School of Electrical and Computer Engineering

Summary of Background Module

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Power

Frequency Analysis
Important Concepts and Skills

**CHARGE AND CURRENT**
- Understand charge and current
- Be able to calculate the effect of one charge on other charges
- Be able to calculate current from charge flow in time

**VOLTAGE**
- Understand how voltage is created by differences in charge density
- Understand how chemical reactions can cause a voltage
- Be able to use voltage references

**POWER AND ENERGY**
- Be able to calculate power from energy
- Be able to analyze circuit using conservation of power
- Be able to analyze circuit using power, voltage, and current

**CIRCUIT DIAGRAMS**
- Be able to identify some basic circuit elements
- Be able to identify nodes of a circuit diagram
- Be able to identify open/short circuits
- Be able to move elements of a diagram