Linear Circuits

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An introduction to linear electric components and a study of circuits containing such devices.

Concept Map

1 Background

2 Resistive Circuits

3 Reactive Circuits

4 Frequency Analysis

5 Power
Resistive vs Reactive Circuit

Concept Map

- Background
  - Current, voltage, sources, resistance
- Methods to obtain circuit equations (KCL, KVL, mesh, node, Thévenin)
- Resistive Circuits
- Power
- Frequency Analysis
  - Applications
  - RLC Circuits
  - 2-order Diff Eqn Equations
  - Differential
  - Inductors
  - RC Circuits
  - Capacitors
  - RL Circuits
Capacitance

• Describe the behavior of capacitors by calculating:
  • the charge stored on the capacitor plates
  • the current flowing through the capacitor
  • the voltage across the capacitor
  • the capacitance of the capacitor

Lesson Objectives

- Describe the construction of a capacitor
- Find charge stored on a capacitor
- Find the current through a capacitor
- Find the voltage across a capacitor
- Calculate the capacitance of a capacitor
- Explain how current flows “through” a capacitor
Capacitors

$$v$$

$$E$$

Capacitors and Charge

$$q = Cv$$
### Current and Voltage

\[ i = C \frac{dv}{dt} \quad v = \frac{1}{C} \int_{t_0}^{t} i(\tau) d\tau + v(t_0) \]

<table>
<thead>
<tr>
<th>Capacitance</th>
<th>Units</th>
<th>Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>farad (F)</td>
<td>C</td>
</tr>
</tbody>
</table>

### Calculating Capacitance

\[ C = \frac{\varepsilon \omega l}{d} \]

\[ \varepsilon = \varepsilon_r \varepsilon_0 \]

\[ \varepsilon_0 \approx 8.85 \times 10^{-12} \text{ F/m} \]
Permittivity of Common Materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Approximate $\varepsilon_r$ (or $k$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>1</td>
</tr>
<tr>
<td>Teflon</td>
<td>2.1</td>
</tr>
<tr>
<td>Paper</td>
<td>3.9</td>
</tr>
<tr>
<td>Glass</td>
<td>4.7</td>
</tr>
<tr>
<td>Rubber</td>
<td>7.0</td>
</tr>
<tr>
<td>Silicon</td>
<td>11.7</td>
</tr>
<tr>
<td>Water</td>
<td>78.5 (varies by T)</td>
</tr>
</tbody>
</table>

Current “Through” A Capacitor
Summary

- Identified how capacitors work
- Calculated charge stored on a capacitor
- Identified the relationship between current and voltage on a capacitor
- Calculated capacitance
- Explained how current flows “through” a capacitor
Capacitors

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• Present how capacitors work in a system
• Identify behavior in DC circuits
• Graphically represent the relationships between current, voltage, power, and energy

Lesson Objectives

- Analyzing capacitors in series/parallel
- Analyze DC circuits with capacitors
- Calculate energy in a capacitor
- Sketch current/voltage/power/energy curves
Capacitors in Parallel

\[ i = i_1 + i_2 \]
\[ = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} \]
\[ = (C_1 + C_2) \frac{dv}{dt} \]

Capacitors in Series

\[ v_{ab} = v_1 + v_2 \]
\[ = \frac{1}{C_1} \int_0^t i(\tau) d\tau + v_1(0) + \frac{1}{C_2} \int_0^t i(\tau) d\tau + v_2(0) \]
\[ = \left( \frac{1}{C_1} + \frac{1}{C_2} \right) \int_0^t i(\tau) d\tau + v_1(0) + v_2(0) \]
\[ \frac{dv_{ab}}{dt} = \left( \frac{1}{C_1} + \frac{1}{C_2} \right) i(t) \]
\[ i(t) = \left( \frac{1}{C_1} + \frac{1}{C_2} \right)^{-1} \frac{dv_{ab}}{dt} \]
Behavior in DC Circuits

Stored Energy

\[ p(t) = i(t)v(t) \]
\[ w(t) = \int_{t_0}^{t} p(\tau)d\tau + w(t_0) \]
\[ i = C \frac{dv}{dt} \]
\[ w = \int_{t_0}^{t} C v(\tau) \frac{dv(\tau)}{d\tau} d\tau + w(t_0) \]
\[ w = \int_{V(t_0)}^{V(t)} C vdv + w(t_0) \]
\[ w = \frac{1}{2} C v^2(t) \bigg|_{V=v(t_0)}^{V(t)} + w(t_0) \]

\[ w = \frac{1}{2} C v^2(t) \]
Graphs

Summary

- Calculated capacitance for capacitors in parallel/series configurations
- Identified how capacitors in DC circuits behave like open circuits
- Derived an equation for the energy stored by a capacitor as an electric field
- Showed graphically the relationships between voltage/current/power/energy in capacitors
Inductance

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• Introduce inductors and describe how they work
• Calculate current and voltage for inductors

Lesson Objectives

- Describe the construction and behavior of an inductor
- Find current through an inductor
- Find voltage across an inductor
- Explain how a voltage is created across an inductor
**Inductors**

![Inductor Diagram]

**Current and Voltage**

\[ v(t) = L \frac{di}{dt} \]

\[ i = \frac{1}{L} \int_{t_0}^{t} v(\tau) d\tau + i(t_0) \]

<table>
<thead>
<tr>
<th>Inductance</th>
<th>Units</th>
<th>Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>henry (H)</td>
<td>L</td>
</tr>
</tbody>
</table>
Ampère’s Law

How Inductors Work
Voltages Across a Wire

\[ i \]

\[ +v \quad - \]

Summary

- Presented the equations for current and voltage in inductors
- Introduced Ampère’s Law and showed how inductors work in context of this law
- Showed how a voltage is created across an inductor as currents change in a system
Inductors

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- Present how inductors work in a system
- Identify behavior in DC circuits
- Graphically represent the relationships between current, voltage, power, and energy

Learning Objectives

- Analyze inductors in series/parallel
- Analyze DC circuits with inductors
- Calculate the energy in an inductor
- Sketch current/voltage/power/energy curves
Inductors in Series

\[ v = v_1 + v_2 \]
\[ = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} \]
\[ = (L_1 + L_2) \frac{di}{dt} \]

Inductors in Parallel

\[ i = i_1 + i_2 \]
\[ = \frac{1}{L_1} \int_{t_0}^{t} v(\tau)d\tau + i_1(t_0) + \frac{1}{L_2} \int_{t_0}^{t} v(\tau)d\tau + i_2(t_0) \]
\[ = \left( \frac{1}{L_1} + \frac{1}{L_2} \right) \int_{t_0}^{t} v(\tau)d\tau + i_1(t_0) + i_2(t_0) \]
\[ v(t) = \left( \frac{1}{L_1} + \frac{1}{L_2} \right)^{-1} \frac{di(t)}{dt} \]
Behavior in DC Circuits

\[ p(t) = i(t)v(t) \quad w(t) = \int_{t_0}^{t} p(\tau) d\tau + w(t_0) \quad v = L \frac{di}{dt} \]

\[ w = \int_{t_0}^{t} Li(\tau) \frac{di(\tau)}{d\tau} d\tau + w(t_0) \]

\[ w = \int_{i(t_0)}^{i(t)} Lidi + w(t_0) \]

\[ w = \frac{1}{2} Li^2(t) \quad w = \frac{1}{2} Li^2(t)|_{i(t_0)} + w(t_0) \]
Graphs

Summary

- Calculated inductance for inductors in parallel/series configurations
- Identified how inductors in DC circuits behave like short circuits
- Derived an equation for the energy stored by an inductor as a magnetic field
- Showed graphically the relationships between voltage/current/power/energy in inductors
First-Order Differential Equations

Solve and graph solutions to first-order differential equations

Lesson Objectives

Examine first-order differential equations with a constant input

- Write the solution
- Sketch the solution
Ordinary Differential Equations

- ODE: Include functions of variables and their derivatives.

\[
\begin{align*}
\frac{dy}{dt} + 2y &= 4 \\
\frac{dy}{dt} - 2y &= 4 \sin(\omega t) \\
\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 4y &= f(t) \\
\frac{dv}{dt} + 2v &= i(t)
\end{align*}
\]

Models of Physical Systems

- Inputs, \( f(t) \)
- Outputs, \( y(t) \)
- \( \frac{dy}{dt} + ay = f(t) \)
Solution to First-Order Differential Equation

\[ \frac{dy}{dt} + ay = K, \quad y(0) \]

Has solution:

\[ y(t) = \frac{K}{a} (1 - e^{-at}) + y(0)e^{-at}, \quad t \geq 0 \]

If \( a \geq 0, \ e^{-at} \to 0 \) \( \Rightarrow y(t) \to \frac{K}{a} = \text{steady-state} \)

Graph of Response

\[ y(t) = \frac{K}{a} (1 - e^{-at}) + y(0)e^{-at}, \quad t \geq 0 \]

STEADY-STATE \[ \frac{K}{a} \]

TRANSIENT \[ (y(0) - \frac{K}{a})e^{-at}, \quad t \geq 0 \]
**Time Constant**

**TIME CONSTANT** — time, $\tau$, for exponential transient to decay to $e^{-1} \approx 0.37$ of its initial value (or 63% to its final value)

**Sample Problems**
Summary

- Discussed how various physical phenomena are modeled by differential equations
- Showed the solution to a generic first-order differential equation with a constant input and initial condition
- Introduced the transient and steady-state responses
- Showed how to sketch the response and plot the time constant
RC Circuits

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- Generate a differential equation that describes the behavior of a circuit with resistors and capacitors
- Solve the differential equation for step inputs (or switching constant inputs)
- Graph the behavior

Lesson Objectives

- Generate a differential equation from a circuit
- Identify initial and final conditions
- Solve the differential equation
- Graph the result
Behavior of RC circuits

Example 1: Initial and Final Conditions

\( v_s = 5V \)
\( R = 2k\Omega \)
\( C = 500\mu F \)
Example 1: Differential Equation

\[ R = 2k\Omega \]
\[ v_s = 5V \]
\[ t = 0 \]
\[ C = 500\mu F \]

\[ i = C \frac{dv_c}{dt} \]
\[ i = \frac{v_s - v_c}{R} \]
\[ \frac{v_s - v_c}{R} = C \frac{dv_c}{dt} \]

\[ \frac{dv_c}{dt} + \frac{1}{RC} v_c = \frac{v_s}{RC} \]
Let \( \tau = RC \)

\[ \frac{dy}{dt} + ay = K \]
\[ y(0) \]
\[ y(t) = \frac{K}{a} (1 - e^{-at}) + y(0)e^{-at}, \quad t \geq 0 \]

\[ v_c = v_s \left( 1 - e^{-\frac{t}{\tau}} \right) \]

Example 1: Graph

\[ v_c = v_s \left( 1 - e^{-\frac{t}{\tau}} \right) \]
Example 2: Initial and Final Conditions

\[ i_s = 1 \text{mA} \quad R_1 = 12 \text{k}\Omega \quad C = 1 \text{pF} \quad R_2 = 4 \text{k}\Omega \]

Example 2: Differential Equation

\[ v = i_s R_{eq} \left( 1 + e^{-\frac{t}{\tau}} \right) + i_s R_1 e^{-\frac{t}{\tau}} \]

\[ i_s = i_{R_1} + i_C + i_{R_2} \]

\[ i_c = C \frac{dv}{dt} \quad i_{R_1} = \frac{v}{R_1} \quad i_{R_2} = \frac{v}{R_2} \]

\[ i_s = \left( \frac{1}{R_1} + \frac{1}{R_2} \right) v + C \frac{dv}{dt} \]

Let \( R_{eq} = \left( \frac{1}{R_1} + \frac{1}{R_2} \right)^{-1} \)

\[ \frac{dx}{dt} = \frac{1}{R_{eq} C} \]

Let \( \tau = R_{eq} C \)
Example 2: Graph

\[ v = 3(1 + e^{-\frac{t}{\tau}}) + 12e^{-\frac{t}{\tau}} \]

Summary

- Got some intuition about how RC circuits behave
- Identified initial and final conditions
- Found differential equations for the circuit and solved them
- Graphed the results
Module 3
Lab Demo:
RC Circuits

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Summary of Reactive Circuits Module

RC Circuit

\[ V \leftrightarrow R \leftrightarrow C \leftrightarrow V_c \leftrightarrow V_s \leftrightarrow R \leftrightarrow C \leftrightarrow V_c \]
RC Circuit

![RC Circuit Diagram]

RC Circuit

![RC Circuit Diagram]
Lab Demo: RC Circuits

Summary

- An oscilloscope is used to measure and record voltage signals versus time.
- A function generator allows you to input voltage signals into a circuit.
- Inputting a square wave into a circuit allows you to capture RC circuit transient behavior and to measure the time constant.
RL Circuits

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- Use differential equations to show the behavior of an RL circuit as the system changes.

Lesson Objectives

- Generate a differential equation from a circuit
- Identify initial and final conditions
- Solve the differential equation
- Graph the result
Behavior of RL circuits

Example 1: Initial and Final Conditions
**Example 1: Differential Equation**

\[ i_L = i_s \left[ 1 - e^{-t/\tau} \right] \]

\[ v_L = i_s R e^{-t/\tau} \]

\[ v_L = L \frac{di_L}{dt} \quad i_R = \frac{v}{R} \quad i_s = i_R + i_L \]

\[ i_s = \frac{L}{R} \frac{di_L}{dt} + i_L \]

\[ \frac{di_L}{dt} + \frac{R}{L} i_L = \frac{R}{L} i_s \quad \text{Let } \tau = \frac{L}{R} \]

**Example 1: Graph**

\[ v_L = 5e^{-t/\tau} \]
Example 2: Initial and Final Conditions

\[ v_1 = 12V \quad v_2 = -3V \quad R_1 = 3k\Omega \quad R_2 = 6k\Omega \quad L = 3mH \]

Example 2: Differential Equation

\[ i_L = \frac{v_2}{R_2} \left[ 1 - e^{-\frac{t}{\tau}} \right] + \frac{v_1}{R_1} e^{-\frac{t}{\tau}} \]

\[ v_L = L \frac{di_L}{dt} \quad i_L = \frac{v_2 - v_L}{R_2} \]

\[ \frac{di_L}{dt} + \frac{R_2}{L} i_L = \frac{v_2}{L} \quad \text{Let } \tau = \frac{L}{R_2} \]
Example 2: Graph

\[ i_L = -\frac{1}{2} \left[ 1 - e^{-\frac{t}{\tau}} \right] + 4e^{-\frac{t}{\tau}} \]

Summary

- Got some intuition about how RL circuits behave
- Identified initial and final conditions
- Found differential equations for the circuit and solved them
- Graphed the results
Lesson Objectives

Examine second-order differential equations with a constant input:
- Determine the steady-state solution
- Determine the type of transient response
- Recognize the characteristics of the plot of the solution
Ordinary Differential Equations

- ODE: Include functions of variables and their derivatives

\[ \frac{d^2y}{dt^2} + 2 \frac{dy}{dt} + 4y = f(t) \]

Models of Physical Systems

- Vibratory System
  - Inputs, \( f(t) \)
  - Outputs, \( y(t) \)
  - System Model: \( \frac{d^2y}{dt^2} + 2 \frac{dy}{dt} + 4y = f(t) \)

- RLC Circuit
  - Voltage or current source
  - Voltage or current

System Model
Solutions to Second-Order Differential Equation

\[ \frac{d^2y}{dt^2} + a_1 \frac{dy}{dt} + a_2 y = K, \quad y(0) \text{ and } \left. \frac{dy}{dt} \right|_{t=0} \]

Solution: \( y(t) = \text{steady-state} + \text{transient} \)

**STEADY-STATE:** \( y(t) \rightarrow \frac{K}{a_2} \)

**TRANSIENT:** Three possible forms depending on roots of \((s^2+a_1s+a_2)=0.\)

Transient Response

\((s^2+a_1s+a_2)=0.\)

**OVERDAMPED:** ((real and distinct roots, \(r_1\) and \(r_2\))

\[ K_1 e^{r_1t} + K_2 e^{r_2t} \]

**CRITICALLY DAMPED:** ((real and equal roots, \(r\) and \(r\))

\[ K_1 e^{rt} + K_2 te^{rt} \]

**UNDERDAMPED:** ((complex roots, \(a+jb\))

\[ Ke^{at} \sin(bt+\varphi) \]
Sample Problems

Overdamped

\[
\frac{d^2 y}{dt^2} + 5 \frac{dy}{dt} + 4y = 1, \quad y(0) = 0, \quad \frac{dy}{dt} \bigg|_{t=0} = 0
\]

Sample Problems

Underdamped

\[
\frac{d^2 y}{dt^2} + 0.8 \frac{dy}{dt} + 4y = 1, \quad y(0) = 0, \quad \frac{dy}{dt} \bigg|_{t=0} = 0
\]
Summary

- Examined generic 2\textsuperscript{nd} order differential equation
  - Vibratory systems, RLC circuits
- Showed steady-state solution
- Showed generic transient solutions to underdamped and overdamped responses
- Showed characteristic plots of underdamped and overdamped responses to a constant input applied at t=0
RLC Circuits Part 1

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- Use differential equations to show the behavior of an RLC circuit as the system changes.

Lesson Objectives

- Generate a second-order differential equation from a RLC circuit
- Identify initial and final conditions
- Solve the differential equation
- Recognize if a system is underdamped/overdamped
Example 1: Initial and Final Conditions

\[ v_s = 5V \]

\[ v_C(0^-) = 0 \]

\[ R = 20k\Omega \quad L = 3.3mH \quad C = 0.01\mu F \]

Example 1: Differential Equation

\[ LC \frac{d^2 v_C}{dt^2} + RC \frac{dv_C}{dt} + v_C = v_s \]

\[ \frac{d^2 v_C}{dt^2} + R \frac{dv_C}{dt} + \frac{1}{LC} v_C = \frac{v_s}{LC} \]

\[ \frac{d^2 y}{dt^2} + a_1 \frac{dy}{dt} + a_2 y = K \]

\[ v_s = v_R + v_L + v_C \]

\[ v_R = iR \quad v_L = L \frac{di}{dt} \quad i = C \frac{dv_C}{dt} \]
**Example 1: Transient**

\[
\frac{d^2v_c}{dt^2} + \frac{R}{L} \frac{dv_c}{dt} + \frac{1}{LC} v_c = \frac{v_s}{LC}
\]

\[
\frac{d^2v_c}{dt^2} + 6.06e6 \frac{dv_c}{dt} + 30.3e9v_c = 151.5e9
\]

**Characteristic Equation:**

\[s^2 + 6.06e6 \ s + 30.3e9 = 0\]

**Roots:** \[-5.00e3 \quad -6.06e6\]

\[v_{c,t} = K_1 e^{-5.00e3t} + K_2 e^{-6.06e6t}\]

---

**Example 1: Steady State**

\[
\frac{d^2v_c}{dt^2} + \frac{R}{L} \frac{dv_c}{dt} + \frac{1}{LC} v_c = \frac{v_s}{LC}
\]

\[v_c \rightarrow \frac{K}{a_2} = LC \frac{v_s}{LC} = v_s\]

\[v_{c,s} = 5\]
Example 1: Solving for Constants

\[ v_c = v_{c,t} + v_{c,s} \]

\[ v_c = K_1 e^{-5.00e3t} + K_2 e^{-6.06e6t} + 5 \]

\[ v_i = 5V \]

\[ i = 5 \mu A \]

\[ R = 20k\Omega \]

\[ L = 3.3mH \]

\[ C = 0.01\mu F \]

\[ K_1 = -5.00413 \]

\[ K_2 = 0.00413 \]

Example 1: Final Solution

\[ v_C = -5.00413e^{-5.00e3t} + 0.00413e^{-6.06e6t} + 5 \]
Summary

- Looked at an overdamped case
- Identified initial and final conditions
- Found and solved representative differential equation
- Plotted the results
RLC Circuits
Part 2

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• Use differential equations to show the behavior of an RLC circuit as the system changes.

Lesson Objectives

- Generate a second-order differential equation from an underdamped RLC circuit
- Identify initial and final conditions
- Solve the differential equation
- Recognize if a system is underdamped/overdamped
- Identify the effect of damping on a second-order system
Example 2: Initial and Final Conditions

\[ v_s = 5V \]

\[ R = 100\Omega \quad L = 3.3\text{mH} \]

\[ C = 0.01\mu F \]

\[ v_C \]

\[ i \]

Example 2: Differential Equation

\[ s^2 + 30.3e3s + 30.3e9 = 0 \]

Roots: \[ -15.15e3 \pm j173.4e3 \]

\[ v_{c, t} = Ke^{-15.15e3t} \cos(173.4e3t + \theta) \]

\[ v_{c, s} = 5 \]
**Example 2: Final Solution**

\[ v_C = 5.019e^{-15.15e^3t} \cos(173.4e^3t - 4.99^o) \]

\[ v_s = 5V \]

\[ R = 100\Omega \quad L = 3.3mH \quad C = 0.01\mu F \]

**Damping Ratio**

\[ \frac{d^2y}{dt^2} + 2\zeta\omega_n \frac{dy}{dt} + \omega_n^2 y = K \]

- \( \zeta > 1 \) Overdamped
- \( \zeta = 1 \) Critically damped
- \( \zeta < 1 \) Underdamped

\[ \omega_n = \sqrt{\frac{1}{LC}} \quad \zeta = \frac{R}{2\omega_n L} \]

\[ R = 20000\Omega \quad \zeta = 17.4078 \]

Effect of Damping Ratio (\( \zeta \))

\[ \text{Time (s)} \times 10^3 \]
Summary

- Got some intuition about how RLC circuits behave and contrasted overdamped and underdamped cases
- Identified initial and final conditions
- Found and solved representative differential equations
- Plotted the results
- Animated response as the resistance changes to show the effect of damping on the system
Lab Demo: RLC Circuit

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Transient response of an RLC circuit

RLC Circuit Measurements

[RLC circuit diagram with labels: 3.3mH, 20kΩ, 0.01μf, V_s, V_c]
Lab Demo: RLC Circuits

Summary

- An underdamped RLC circuit has a small R value and results in large peaks
- An overdamped RLC circuit has a large R value
Lab Demo: Applications of Capacitance

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Show common applications of capacitance

Applications of Capacitance

\[ C = \frac{\varepsilon WL}{d} \]
Lab Demo: Applications of Capacitance

Summary

- Showed capacitive sensors such as touch pads and capacitive microphone, and antenna tuner
Credits

Thanks to Allen Robinson, James Steinberg, Kevin Pham, and Al Ferri for help with demonstration ideas.

Thanks to Marion Crowder for videotaping the demonstration.

Capacitance drawings done by Nathan Parrish.
Lab Demo: Applications of Inductance

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Show common applications of inductance

Lab Demo: Applications of Inductance
Summary

- Discussed energy exchange in inductors – mechanical to electrical and vice versa
  - Moving conductor in magnetic field induces current
  - Changing current in coiled wire causes a magnetic field
- Showed inductance applications
  - Passive Sensing (guitar pick-up)
  - Active Sensing (metal detector)
  - Actuation (solenoid, speaker)

Credits

Thanks to Allen Robinson, James Steinberg, Kevin Pham, and Al Ferri for help with demonstration ideas. Thanks to Ken Connor and Don Millard for the guitar string experiment.

Thanks to Marion Crowder for videotaping the demonstration.

Inductance drawings done by Nathan Parrish.
Module 3
Reactive Circuit Wrap Up

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Summary of Reactive Circuits Module

Concept Map

Background
Current, voltage, sources, resistance

Resistive Circuits
Methods to obtain circuit equations (KCL, KVL, mesh, node, Thévenin)

Power

Reactive Circuits
- Capacitors
- RC Circuits
- Inductors
- RL Circuits
- Differential Equations
- 2-order Diff Eqn Equations
- RLC Circuits
- Applications

Frequency Analysis
Important Concepts and Skills

- **CAPACITANCE AND CAPACITORS**
  - Understand the basic structure of a capacitor and its fundamental physical behavior
  - Be able to use the $i-v$ relationship to calculate current from voltage or vice versa
  - Be able to reduce capacitor connections using parallel and series connections
  - Be able to calculate energy in a capacitor
  - Be able to sketch current/voltage/power/energy curves

- **INDUCTANCE AND INDUCTORS**
  - Be able to describe the construction and behavior of an inductor
  - Be able to use the $i-v$ relationship to find current through an inductor from the voltage across it, and vice versa
  - Be able to explain how a voltage is created across an inductor
  - Be able to analyze inductors in series/parallel
  - Be able to calculate the energy in an inductor
  - Be able to sketch current/voltage/power/energy curves
Important Concepts and Skills

- **FIRST-ORDER DIFFERENTIAL EQUATIONS**
  - Given a constant input, be able to determine the steady-state value, time constant, and sketch the response

- **RC CIRCUITS AND RL CIRCUITS**
  - Be able to write a differential equation governing the behavior of the circuit
  - Be able to calculate the time constant, steady-state value, and sketch the response

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Important Concepts and Skills

- **2ND ORDER DIFFERENTIAL EQUATIONS**
  - Be able to identify the steady-state value
  - Be able to predict the type of response from the roots (underdamped, critically damped, overdamped)

- **RLC CIRCUITS**
  - Be able to write the differential equation that governs the behavior
  - Be able to predict the type of response (underdamped, overdamped, critically damped)
  - Be able to compute the damping factor and the resonant frequency
  - Know that the smaller the damping factor, the larger the oscillations
Important Concepts and Skills

**APPLICATIONS**
- Know the purposes of an oscilloscope and a function generator
- Know several applications of inductors and capacitors when they are used with non-electrical components

Concept Map
Reminder

- Do all homework for this module
- Study for the quiz
- Continue to visit the forum