Module 2: Op Amps
Introduction and Ideal Behavior

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Introduce Op Amps and examine ideal behavior
Lesson Objectives

- Introduce Operational Amplifiers
- Describe Ideal Op Amp Behavior
- Introduce Comparator and Buffer Circuits
Operational Amplifiers (Op Amps)

Specialized circuit made up of transistors, resistors, and capacitors fabricated on an integrated chip

![Operational Amplifier Diagram](image)

**Uses:**
- Amplifiers
- Active Filters
- Analog Computers
Op Amps in Circuits

- Active Element: has its own power supply
- Symbol ignores the +/- $V_S$ in the symbol since it does not affect circuit behavior

$V_S = 10V, 15V$
Open Loop Behavior

\[ v_o = A(v_+ - v_-) \]
Comparator Circuit

\[ V_o = \begin{cases} +V_s & \text{if } v_{in} > 0 \\ -V_s & \text{if } v_{in} < 0 \end{cases} \]
Example

\[ C \sin(\omega t) \]

**Diagram:**
- Input voltage: \( V_+ \) and \( V_- \)
- Output voltage: \( V_o \)
- Typical waveform for an op-amp with \( V_o \) at positive and negative saturation levels.
Ideal Op Amp Behavior

\[ i_+ = i_- = 0 \]

\[ v_+ - v_- = 0 \]
Buffer Circuit

\[ V_{in} = V_{o} \]
Summary

- Op amps are active devices that can be used to filter or amplify signals linearly.
- Ideal op amps:
  - $i_+ = i_- = 0$
  - $v_+ - v_- = 0$
- Circuits: comparator and buffer
Remainder of Module 2: Op Amps

- Buffer Circuit
- Basic Amplifier Configurations
- Differentiators and Integrators
- Active Filters
Demonstrate buffer circuit behavior
Lesson Objectives

- Introduce physical op amps in circuits
- Examine Buffer Circuit behavior
Buffer Circuit

- Use to boost power without changing voltage waveform

\[ v_{in} = v_o \]
Example: Without Buffer
Physical Op Amps

$V_S = 15V$

<table>
<thead>
<tr>
<th>Signal</th>
<th>PIN</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_-$</td>
<td>2</td>
</tr>
<tr>
<td>$v_+$</td>
<td>3</td>
</tr>
<tr>
<td>$-V_S$</td>
<td>4</td>
</tr>
<tr>
<td>$v_o$</td>
<td>6</td>
</tr>
<tr>
<td>$+V_S$</td>
<td>7</td>
</tr>
</tbody>
</table>
Example: With Buffer

\[ V_{in} \rightarrow + \rightarrow R \rightarrow + \rightarrow V_o \]

\[ V_{in} \rightarrow + \rightarrow - \rightarrow + \rightarrow + \rightarrow V_o \rightarrow + \rightarrow R \rightarrow + \]
Example: With Buffer
Summary

- Buffers boost the power without changing the voltage waveform
- Demonstrated physical op amp circuits
Basic Op Amp Amplifier Configurations

*Introduce Inverting and Non-Inverting Amplifiers, Difference and Summing Amplifiers*
Lesson Objectives

- Introduce
  - Inverting and Non-Inverting Configurations
  - Difference and Summing Configurations
- Introduce the Gain of a circuit
Non-Inverting Amplifiers

\[ V_o = \frac{R_2 + R_3}{R_3} V_{in} \]

\[ V_o = G V_{in} \quad \text{Gain}: G = \frac{R_2 + R_3}{R_3} \]
Non-Inverting Amplifier Example

If $R_2 = R_3 = 200\Omega$,

- Since $G > 1$, the input is amplified
- If $G < 1$, the input is attenuated
Inverting Amplifier

\[ V_o = -\frac{R_f}{R_1} V_{in} \]

\[ V_o = G V_{in} \]
Inverting Amplifier Example

R₁ = 1000Ω, Rᶠ = 2000Ω

- If G > 1, the input is amplified
- If G < 1, the input is attenuated
Difference Circuit

\[ V_o = \frac{R_F}{R_1} (V_2 - V_1) \]
Difference Circuit

\[ V_o = \frac{R_F}{R_1} (V_2 - V_1) \]
Summing Amplifier

\[ V_o = G_1 V_1 + G_2 V_2 \]

\[ G_1 = -\frac{R_F}{R_1} \quad G_2 = -\frac{R_F}{R_2} \]
Summary

- Gain: \( V_o = G V_{in} \)

- Amplifier Circuit Configurations
  - Non-Inverting Amplifier
  - Inverting Amplifier
  - Difference Amplifier
  - Summing Amplifier
Differentiators and Integrators

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Introduce Integrating and Differentiating Op Amp Circuits
Lesson Objectives

- Introduce Differentiators and Integrators
- Demonstrate the performance of both circuits on an oscilloscope
Differentiator Circuit

\[
\frac{dv_c}{dt} = \frac{i}{C} = \frac{V_{o}}{RC}
\]

\[
V_o = -RC \frac{dV_{in}}{dt}
\]
Differentiator Circuit

Derivation:
1. KVL: \( V_{\text{in}} = V_c + Ri + V_o \)
2. \( V_{\text{in}} = V_c \)
3. \( V_o = -Ri = -RC \frac{dV_{\text{in}}}{dt} \)
Differentiator Example

\[ V_{in} \to 1 \mu F \to 1000\Omega \to V_{o} \]

\[ +V_{S} = 15v \]
\[ -V_{S} = -15v \]

\[ V_{in} \to v_{-} \to v_{+} \]
\[ v_{o} \]

\[ v_{o} + V_{S} \]

\[ v_{-} \]
Results

\[ V_o = -RC \frac{dV_{in}}{dt} \]
Integrator Circuit

\[ i = C \frac{dV_c}{dt} \]

\[ V_c = \frac{1}{C} \int_0^t i dt \]

\[ V_o = \frac{-1}{RC} \int_0^t V_{in} dt \]
Integrator Circuit

For $t < 0$: $V_{in} = iR$ and $V_o = 0$

For $t > 0$: $V_{in} = iR$

$i = V_{in}/R$

$V_{in} = iR + V_c + V_o$

$V_o = -V_c = -1/C \int_0^t V_{in}/R \, dt$

Derivation:

For $t < 0$: $V_{in} = iR$ and $V_o = 0$

For $t > 0$: $V_{in} = iR$

$i = V_{in}/R$

$V_{in} = iR + V_c + V_o$

$V_o = -V_c = -1/C \int_0^t V_{in}/R \, dt$
Integrator Example

Vin \quad v_+ \quad vo

+V_S = 15v

- V_S = -15v
Results

\[ V_o = \frac{-1}{RC} \int_0^t V_{in} \, dt \]
Summary

- Differentiator and Integrator Op Amp circuits examined
Active Filters

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Introduce active filters and show different types of filters
Lesson Objectives

- Introduce active filter circuits
Analog Filters

Analog Filter $H(\omega)$

Magnitude $|H(\omega)|$

$V_{in}$

$V_{out}$

$\omega$ (rad/sec)

Magnitude

$0 \leq \omega \leq 1000$

$0 \leq v(t) \leq 2$

Time (sec)

$0 \leq t \leq 0.25$
Quiz

\[ V_{in} = 1 + \cos(10(2\pi t)) + \cos(100(2\pi t)) \]

\[ V_{out} = 0.45\cos(10(2\pi t)+\theta_1) + 0.97\cos(100(2\pi t)+\theta_2) \]
Summary of RC and RLC (Passive) Filters

**RC Lowpass:**
\[ V_{\text{in}} \rightarrow R \rightarrow C \rightarrow V_{o} \]

**RC Highpass:**
\[ V_{\text{in}} \rightarrow C \rightarrow R \rightarrow V_{o} \]

**RLC Lowpass:**
\[ V_{\text{in}} \rightarrow L \rightarrow R \rightarrow C \rightarrow V_{o} \]

Bode Plots

- Magnitude (dB) vs. \( \omega \)
- "Magnitude (dB)" vs. \( \omega \)
Limitations of RLC Passive Filters

- Depletes power
- No isolation
Active Filters

Active – has its own power supply
- Most common active filters are made from op amps
- Provide isolation

<table>
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<tr>
<th>Active Filters</th>
<th>Op Amp Circuit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vin</td>
<td>Vout</td>
</tr>
</tbody>
</table>

![Op Amp Circuit Diagram](image-url)
Summary

- An **analog filter** is a circuit that has a specific shaped frequency response.
- A **active filter** is made of op amps and has its own power supply. Advantages over RLC passive filters:
  - Provides isolation (cascade filters)
  - Boosts the power
  - Can provide sharper roll-off
Impedance Gain

Derivation: \( V_{in} = iZ_1 \)

\[ V_o = -iZ_f = -(Z_f/Z_1)V_{in} \]
First-Order Lowpass Filters

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Introduce lowpass filters
Lesson Objectives

- Introduce active lowpass filters
Lowpass Filters

- Lowpass filters pass low frequency components and attenuate high frequency components

Transfer Function $H(\omega)$

**Linear Plot**

- $K_{DC}$
- $0.707K_{DC}$
- $\omega_B$
- $\omega$

**Bode Plot**

- $20\log_{10}(K_{DC})$
- $3dB$
- $\omega$

Magnitude (dB)
First-Order Filter

\[
H(\omega) = K_{DC} \frac{1}{\tau j\omega + 1}
\]

- Bandwidth, \( \omega_B = 1/\tau \)
- DC Gain = \( H(0) = K_{DC} \)

Linear Plot

Magnitude

0 \[\omega_B\] \[\omega\]

0.707\(K_{DC}\)
From Passive to Active Lowpass Filters

Isolation at the input:

Isolation in the output:
First-Order Inverting Lowpass Filter

\[ V_o = -\frac{R_f}{R_1 R_f C j\omega + 1} V_{in} \]
Frequency Characteristics of LP Filter

\[ H(\omega) = -\frac{R_f}{\omega} \frac{1}{R_1 \left( R_f C j \omega + 1 \right)} \]

\[ |H(\omega)| = \frac{R_f}{\omega} \frac{1}{R_1 \sqrt{(R_f C \omega)^2 + 1}} \]

\[ \angle H(\omega) = 180 - \arctan(R_f C \omega) \]

DC Gain = \(-\frac{R_f}{R_1} \)

Bandwidth, \( \omega_b = \frac{1}{R_f C_f} \)
Derivation: Lowpass Filter
Example

Design an inverting lowpass filter to have a DC gain of -2 and a bandwidth of 500 rad/s:

\[ H(\omega) = -\frac{R_f}{R_1} \frac{1}{1 + R_f C j \omega} \]
Summary

- A **lowpass filter** passes low frequency signals and attenuates high frequency signals.

- Three first-order lowpass configurations:
  - Noninverting, isolation at the input
  - Noninverting, isolation at the output
  - Inverting, isolation at input and output
First-Order Highpass Filters

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Introduce highpass filters
Lesson Objectives

- Introduce active highpass filters
Highpass Filter

- Passes high frequency components and attenuates low frequency components

![Linear Plot](image)

![Bode Plot](image)
First-Order Filter

Linear Plot

\[ H(\omega) = \frac{Kj\omega}{\tau j\omega + 1} \]

Corner Frequency, \( \omega_c = 1/\tau \)
Passband Gain= \( K_{PB} = K/\tau \)
Inverting Highpass Filter Configuration

\[ V_o = \frac{-R_f C j \omega}{(R_1 C j \omega + 1)} V_{in} \]
**Frequency Characteristics of HP Filter**

$$H(\omega) = \frac{-R_f C j \omega}{(R_1 C j \omega + 1)}$$

$$|H(\omega)| = \frac{R_f C \omega}{\sqrt{(R_1 C \omega)^2 + 1}}$$

$$\angle H(\omega) = -90^\circ - \arctan(R_1 C \omega)$$

- **Passband Gain** \((\omega \rightarrow \infty) = -\frac{R_f}{R_1}\)
- **Corner Frequency** \(\omega_c = \frac{1}{R_1 C}\)

![Graph showing frequency response and gain characteristics of the HP filter.](graph.png)
Example

Design a highpass filter to have a passband gain of 2 and a corner frequency of 1k rad/s:
Summary

- **A highpass filter** passes high frequency components in signals and attenuates low frequency components.
- First-order highpass filter
  
  ![First-order highpass filter diagram](image)

  $$ H(\omega) = \frac{-R_f C j \omega}{R_1 C j \omega + 1} $$

- Design based on
  - Corner frequency of the passband, $\omega_c$
  - Passband gain, $K_{PB}$